Hong Kong Mathematics Olympiad (1996 – 97)

Final Event 1 (Individual)

香港数学竞赛 (1996 – 97)

决赛项目 1 (个人)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特别声明，答案须用数字表达，并化至最简。

(i) Given that $\frac{3}{a}+\frac{1}{u}=\frac{7}{2}$ and $\frac{2}{a}-\frac{3}{u}=6$ are simultaneous equations in a and u. Solve for a.

$a =$

已知 $\frac{3}{a}+\frac{1}{u}=\frac{7}{2}$ 及 $\frac{2}{a}-\frac{3}{u}=6$ 为 a 与 u 的联立方程。求 a 的解。

(ii) Three solutions of the equation $px+qy+bz=1$ are $(0, 3a, 1)$, $(9a, -1, 2)$ and $(0, 3a, 0)$. Find the value of the coefficient b.

$b =$

方程 $px+qy+bz=1$ 的根分别为 $(0, 3a, 1)$、$(9a, -1, 2)$ 和 $(0, 3a, 0)$。求系数 b 的值。

(iii) Find c so that the graph of $y = mx + c$ passes through the two points $(b + 4, 5)$ and $(-2, 2)$.

$c =$

若 $y = mx + c$ 的图像经过 $(b + 4, 5)$ 及 $(-2, 2)$ 两点。求 c。

(iv) The solution of the inequality $x^2+5x-2c\le 0$ is $d\le x\le 1$. Find d.

$d =$

不等式 $x^2+5x-2c\le 0$ 的解为 $d\le x\le 1$。求 d。

Hong Kong Mathematics Olympiad (1996 – 97)
Final Event 2 (Individual)
香港数学竞赛 (1996 – 97)
决赛项目 2 (个人)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
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(i) By considering :

$\frac{1^2}{1}=1$, $\frac{1^2+2^2}{1+2}=\frac{5}{3}$, $\frac{1^2+2^2+3^2}{1+2+3}=\frac{7}{3}$, $\frac{1^2+2^2+3^2+4^2}{1+2+3+4}=3$, find a such that $\frac{1^2+2^2+\cdots a^2}{1+2+\cdots a}=\frac{25}{3}$.

考虑：

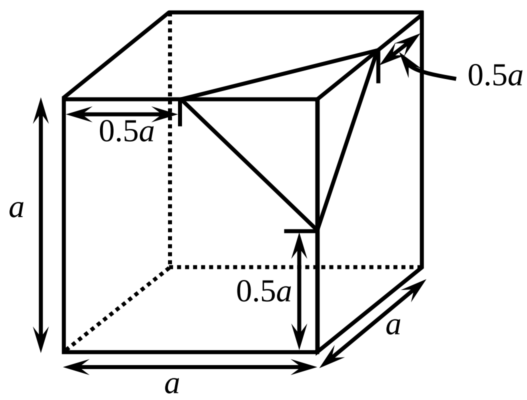
$\frac{1^2}{1}=1$，$\frac{1^2+2^2}{1+2}=\frac{5}{3}$，$\frac{1^2+2^2+3^2}{1+2+3}=\frac{7}{3}$，$\frac{1^2+2^2+3^2+4^2}{1+2+3+4}=3$，求 a 使得 $\frac{1^2+2^2+\cdots a^2}{1+2+\cdots a}=\frac{25}{3}$。

$a =$

(ii) A triangular pyramid is cut from a corner of a cube with side length a cm as the figure shown. If the volume of the pyramid is b cm^3, find b.

$b =$

如图所示，从正立方体的一角割出一个三角椎体。若三角椎体的体积为 b cm^3，求 b。



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(iii) If the value of x^2+cx+b is not less than 0 for all real number x, find the maximum value of c.

$c =$

若对于所有实数 x，x^2+cx+b 不小于 0，求 c 的最大值。

(iv) If the unit digit of 1997^{1997} is $c-d$, find d.

$d =$

若 1997^{1997} 的个位数为 $c-d$，求 d。

Hong Kong Mathematics Olympiad (1996 – 97)
Final Event 3 (Individual)
香港数学竞赛 (1996 – 97)
决赛项目 3 (个人)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
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(i) The average of a, b, c and d is 8. If the average of a, b, c, d and P is P, find P.

a、b、c 和 d 的平均值为 8。若 a、b、c、d 和 P 的平均值为 P，求 P。

$P =$

(ii) If the lines $2x+3y+2=0$ and $Px+Qy+3=0$ are parallel, find Q.

若直线 $2x+3y+2=0$ 和 $Px+Qy+3=0$ 互相平行，求 Q。

$Q =$

(iii) The perimeter and the area of an equilateral triangle are Q cm and $\sqrt{3}R$ cm^2 respectively. Find R.

若等边三角形的周界和面积分别为 Q cm 和 $\sqrt{3}R$ cm^2。求 R。

$R =$

(iv) If $(1+2+\cdots+R)^2=1^2+2^2+\cdots+R^2+S$, find S.

若 $(1+2+\cdots+R)^2=1^2+2^2+\cdots+R^2+S$，求 S。

$S =$

Hong Kong Mathematics Olympiad (1996 – 97)

Final Event 4 (Individual)

香港数学竞赛 (1996 – 97)

决赛项目 4 (个人)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特别声明，答案须用数字表达，并化至最简。

(i) If each interior angle of a n-sided regular polygon is $140°$, find n.

若正 n 边形的内角为 140°，求 n。

$n =$

(ii) If the solution of the inequality $2x^2 - nx + 9 < 0$ is $k < x < b$, find b.

若不等式 $2x^2 - nx + 9 < 0$ 的解为 $k < x < b$，求 b。

$b =$

(iii) If $cx^3 - bx + x - 1$ is divided by $x + 1$, the remainder is -7, find c.

若 $cx^3 - bx + x - 1$ 除以 $x + 1$，余数 为 -7，求 c。

$c =$

(iv) If $x + \frac{1}{x} = c$ and $x^2 + \frac{1}{x^2} = d$, find d.

若 $x + \frac{1}{x} = c$ 和 $x^2 + \frac{1}{x^2} = d$，求 d。

$d =$

Hong Kong Mathematics Olympiad (1996 – 97)

Final Event 5 (Individual)

香港数学竞赛 (1996 – 97)

决赛项目 5 (个人)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
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(i) The volume of a hemisphere with diameter a cm is 18π cm^3, find a.

一直径为 a 的半球体的体积为 18π cm^3，求 a。

$a =$

(ii) If $\sin 10a° = \cos(360° - b°)$ and $0 < b < 90$, find b.

若 $\sin 10a° = \cos(360° - b°)$ 和 $0 < b < 90$，求 b。

$b =$

(iii) The triangle is formed by the x-axis and y-axis and the line $bx + 2by = 120$. If the bounded area of the triangle is c, find c.

一三角形是由 x-轴、y-轴和直线 $bx + 2by = 120$ 所组成。若所包围之三角形的面积为 c，求 c。

$c =$

(iv) If the difference of the two roots of the equation $x^2 - (c+2)x + (c+1) = 0$ is d, find d.

若方程式 $x^2 - (c+2)x + (c+1) = 0$ 两根之差为 d，求 d。

$d =$